RESPIRE: Robust SEnSor Placement OptImization in PRobabilistic Environments

Onat Gungor[1,3], Tajana S. Rosing[2], and Baris Aksanli[3]

[1]Department of Electrical and Computer Engineering, University of California San Diego
[2]Department of Computer Science and Engineering, University of California San Diego
[3]Department of Electrical and Computer Engineering, San Diego State University
ogungor@ucsd.edu, tajana@ucsd.edu, baksanli@sdsu.edu

***Abstract*—Optimal sensor coverage considers where to place sensors at minimal cost while maximizing coverage. This approach often overlooks the robustness of the entire system. If sensors break down, the application performance might severely be affected. This paper constructs a multi-objective optimization model that considers not only optimal coverage, but also robustness. Our method increases the system robustness by up to 50% compared to a coverage-only approach with 201% higher probability of monitoring the entire environment.**

I. Introduction and Related Work

Sensor placement directly impacts the efficiency of the allocated resources and system performance [1], considering coverage and connectivity [2]. For the best performance, applications should observe and monitor the greatest total relevant area possible, i.e. high coverage. Previous studies optimize sensor coverage, with the goal of maximizing sensor coverage with minimum sensor cost [3], [4], energy consumption [5], or communication bandwidth [6]. Solutions to these optimization problems include [7] [8]: 1) Exhaustive search considers all possible locations for sensor placement [9], which has exponential complexity. 2) Optimization-based approaches construct integer programming models which can be solved by conventional solvers, [10]–[12]. but may not obtain a solution in polynomial time. 3) Heuristics try to find nearly optimal solution(s) with reasonable execution time [13]–[17].

When a sensor breaks down, anything in its range can go undetected. Optimizing sensor placement only from a maximum-coverage perspective can lead to a very sparse sensor placement, where most areas are covered by a single sensor. Figure 1 illustrates this issue. The small circles are sensors that can detect up to 1 unit, and the middle point is the point-of-interest (PoI). The top-left figure shows a coverage-only approach where only the sensor at (1,2) can detect the PoI. If this sensor breaks down (top-right), the PoI becomes undetectable. The bottom-left figure is a robust method where all sensors can detect the PoI. If the same sensor breaks down, the PoI stays detectable by two other sensors (bottom-right). Some similar works [3], [18]–[22] place sensors by considering possible sensor failures, but they do not quantify or verify the robustness.

We formulate a robustness-aware sensor placement problem and quantify robustness with a metric called "detectability

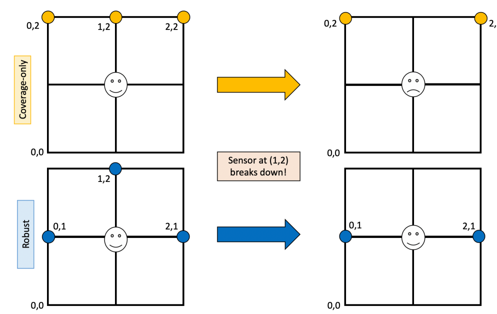


Figure 1: Coverage-only vs. robustness-aware approaches

degree" for each POI, that measures with how much probability a location is covered by the deployed sensor(s). We run experiments with multiple room configurations to compare our approach with the coverage-only method. By placing sensors considering the detectability degrees of all locations, the robustness can be increased by up to 50% compared to a coverage-only approach. We verify this robustness improvement by analyzing system coverage with broken sensors. Our sensor placement has up to 201% higher probability of monitoring the entire environment, compared to a coverage-only approach. Our method leads to a more robust sensor placement, enabling the application to continue to perform effectively even when there are non-functional sensors.

II. Robust Sensor Placement Formulation

We represent the area to cover as a 3-D grid. Sensors can be placed at the grid points and all points should be covered. The sensing range r indicates the maximum distance a sensor can cover. A sensor at location (x_1, y_1, z_1) can cover a target at (x_2, y_2, z_2) if and only if r is greater than or equal to *Euclidean distance* between the sensor and the target.

Coverage-only Base Model: We reformulate the coverage-only problem using maximal covering location problem [23], with the below parameters:

- $\mathcal{N}$ = number of sensors to be located
- $\mathcal{G}$ = set of grid points to be detected
- $\mathcal{S}$ = set of potential sensor locations
- g = index of grid point $g \in \mathcal{G}$
- s = index of possible sensor location $s \in \mathcal{S}$

r = sensor sensing range
d_{sg} = euclidean distance between sensor and grid point
$\xi_{sg} = 1$ if $d_{sg} \leq r; 0$ otherwise
The decision variables are as follows:
$X_s = 1$ if sensor is positioned at location s; 0 otherwise
$Y_g = 1$ if grid point g is detected; 0 otherwise
The integer linear programming (ILP) model becomes:

$$\text{maximize} \quad \sum_{g \in \mathcal{G}} Y_g \tag{1}$$

$$\text{subject to} \quad \sum_{s \in \mathcal{S}} \xi_{sg} X_s \geq Y_g \quad \forall g \in \mathcal{G} \tag{2}$$

$$\sum_{s \in \mathcal{S}} X_s = \mathcal{N} \tag{3}$$

$$X_s = \{0, 1\} \quad \forall s \in \mathcal{S}, \quad Y_g = \{0, 1\} \quad \forall g \in \mathcal{G} \tag{4}$$

(1) is the objective function which maximizes the number of grid points covered. Constraints (2) enable a grid point g to be covered if and only if one or more sensors can detect it. Constraint (3) forces to place exactly $\mathcal{N}$ sensors. Constraints (4) are binary variable constraints for the decision variables.

Coverage-only Probabilistic Model: The base coverage-only model assumes that a sensor can only make binary detection decisions. In reality, there is an uncertainty with sensor readings. Thus, sensor detection should be based on a probabilistic model [24], e.g. with respect to the distance between a sensor and a point. To achieve this, we define p_{sg} as the detection probability of a point g by sensor at point s. We use a common function [25] for the relationship between d_{sg} and p_{sg}: $p_{sg} = e^{-\alpha d_{sg}}$ where α denotes the rate at which sensor's detection probability decreases with distance. With larger α, p_{sg} decreases quicker with distance.

We calculate p_{sg} values for all possible sensor-grid point tuples using the above function. We denote the probability of missing a grid point g with a sensor located at s as $1 - p_{sg} X_s$. We use τ_g as the maximum allowable miss probability for each point (between 0 and 1). Larger τ_g leads to full coverage with fewer sensors (flexible system), whereas smaller τ_g requires more sensors to obtain full coverage (strict system). We reformulate constraints (2) as:

$$\sum_{s \in \mathcal{S}} \eta_{sg} X_s \geq \zeta_g Y_g \quad \forall g \in \mathcal{G} \tag{5}$$

where $\eta_{sg} = -\ln(1 - p_{sg})$ and $\zeta_g = -\ln(\tau_g)$. Here, the meaning of Y_g changes from before, where it measures the number of points that satisfy constraints (5).

Robustness-Aware Probabilistic Model: In a sensor network, each sensor may not correctly and accurately function indefinitely. There might be some environmental disruptions which affect the working condition of a sensor, leading to malfunctioning, inaccurate readings, or a complete breaking down. To prevent this, we need to place the sensors in a way to increase the resilience of the system. We construct a robust sensor placement model with probabilistic detection. We define "detectability degree" (δ_g) as the sum of detection probabilities from placed sensors to each point:

$$\delta_g = \sum_{s \in \mathcal{S}} p_{sg} X_s \quad \forall g \in \mathcal{G} \tag{6}$$

To understand this formulation better, consider *Figure* 1 where sensor detection is binary (p_{sg} is 1 if detected, otherwise it is 0). In the top-left figure, δ_g is 1, whereas in bottom-left it is three (i.e. the point can be detected by three sensors.) In our case, instead of binary numbers (0 or 1), we use detection probabilities to find δ_g. We define the robustness of a system using average (μ) and minimum (ψ) δ_g. (10) and (11) provide mathematical formulations for these variables. For a location, higher detectability degree means a more robust system because if some sensor(s) covering that point break down, there are alternatives to cover that particular point.

We provide a simple sensor placement scenario to explain these values in more detail. Assume that we have five points to detect and seven sensors with binary sensor detection, where $\{2, 5, 6, 7, 0\}$ gives us the number of sensors that can detect each point (i.e. first point is detected by two sensors, etc.) In this set, the average detection value is 4 (20/5), with minimum as 0. Even though the average value is high, there are still points with significantly low values, thus making the system vulnerable to sensor break downs. Thus, we need to consider both the average and minimum values to obtain a more equally distributed set. For our formulation, we replace the number of sensors with detection probabilities. Below is the multi-objective optimization model with weighted sums for the average and minimum values [26]:

$$\text{maximize} \quad w_1 \mu + w_2 \psi \tag{7}$$

$$\text{subject to} \quad \sum_{s \in \mathcal{S}} \eta_{sg} X_s \geq \zeta_g \quad \forall g \in \mathcal{G} \tag{8}$$

$$\sum_{s \in \mathcal{S}} X_s = \mathcal{N} \tag{9}$$

$$\mu = \frac{\sum_{g \in \mathcal{G}} \sum_{s \in \mathcal{S}} p_{sg} X_s}{|\mathcal{G}|} \tag{10}$$

$$\psi \leq \sum_{s \in \mathcal{S}} p_{sg} X_s \quad \forall g \in \mathcal{G} \tag{11}$$

$$w_1 + w_2 = 1 \tag{12}$$

(7) is our objective function which maximizes the sum of average and minimum detectablity degrees. Constraints (8) are the probabilistic constraints where $\eta_{sg} = -\ln(1 - p_{sg})$ and $\zeta_g = -\ln(\tau_g)$. Constraint (9) forces to place exactly N number of sensors. Constraint (10) is an equality constraint for average detectability degree where $|\mathcal{G}|$ denotes cardinality of set $\mathcal{G}$. Constraint (11) is another equality constraint which denotes minimum detectability degree. Constraint (12) forces sum of weights to be equal to 1. In our model, we select the values of w_1 and w_2 as 0.5, i.e. we assign equal importance to the average and minimum detectability degrees.

Table I: Minimum (average) value improvement with 1 broken sensor

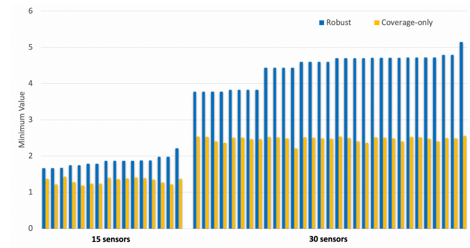
# Sensors	Small	Medium	Large	Very Large
15	38% (2.7%)	67% (5.4%)	33% (5.9%)	26% (7.8%)
20	196% (0.9%)	146% (0.4%)	101% (3.7%)	82% (7.2%)
25	146% (0.5%)	180% (0.8%)	126% (1.7%)	108% (4.9%)
30	79% (-0.2%)	201% (0.8%)	131% (0.7%)	94% (7.4%)

III. Evaluation

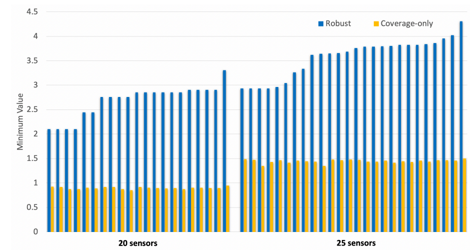
Experimental Setup: We implement ILP models in CPLEX 12.10 [27] and run experiments on a PC with 16 GB RAM and an 8-core 2.3 GHz Intel Core i9 processor. We adopt the setup from [28] with low-resolution thermal sensors. We consider different room configurations with a fixed height of $3m$ [29]. The distance between each point is $1.5m$. Sensors can be placed on each point and all points should be covered by a sensor. Some points are not feasible for sensor placement, e.g. some middle points (as a sensor cannot be placed in the air). The room settings are: 1) small: $4.5m \times 4.5m \times 3m$, 2) medium: $6m \times 6m \times 3m$, 3) large: $7.5m \times 7.5m \times 3m$, 4) very large: $9m \times 9m \times 3m$. We use a τ_g value of 0.4 for the small room and increase it by 0.05 for each larger setting. We experimentally determine these τ_g values to provide a balanced sensor placement. To find the optimal value of α, we perform an experiment to measure the probability of detecting a person with respect to increasing distance. We use curve-fitting on the measured values and obtain the optimal α as 0.576.

Results: The left hand side (LHS) of Equation 8 for each point indicates how well the point is covered. As the LHS gets bigger, the point is covered with higher probability, less prone to sensor break downs. To illustrate robustness, we create a scenario where one sensor breaks down. We calculate the LHS values of Equation 8 for each point for both coverage-only [3], [30] and robustness-aware models. We calculate the minimum and average LHS values across all grid points, excluding the broken sensor. The minimum value shows the most vulnerable point, while the average measures the vulnerability across all points. Figure 2 shows this in detail for the small room with 4 cases, 15 and 30 sensors in a); 20 and 25 sensors in b). For each case, we calculate the minimum LHS value across all points when a particular sensor breaks down, for both coverage-only and robust models. The X-axis indicates different broken sensors, i.e. each blue (robust)/yellow (coverage-only) column pair represents a broken sensor. For each case, the right-most two columns represent no broken sensor case. Our model leads to significantly higher minimum LHS values when a sensor breaks down, i.e. the most vulnerable point with our model has a much higher probability of detection as compared to the coverage-only case, hence less prone to broken sensors.

We expand this analysis on all room settings with 15, 20, 25, and 30 sensors, comparing the minimum (average) LHS value improvement of our method in Table I. The average and minimum value changes are up to 7.8% and 201%, respectively. The minimum value is more important as it shows the most vulnerable point and our method makes the most vulnerable point significantly less prone to broken sensors.



(a) 15 & 30 sensors



(b) 20 & 25 sensors

Figure 2: Small room min detectability degree: Coverage-only [3], [30] vs. Our Robust Model (a) 15-30 sensors (b) 20-25 sensors

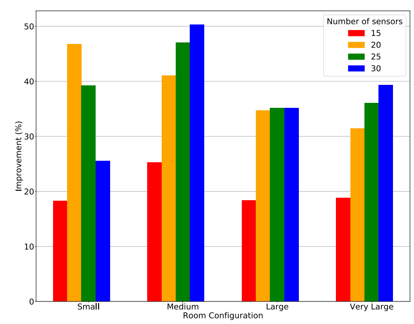


Figure 3: Reliability improvement vs. number of sensors

Next, we calculate Equation 7 to quantify the robustness of a sensor placement, across all room settings with different number of sensors in Figure 3. The maximum robustness improvement is 50% in medium room with 30 sensors. Average improvement is 32%, 41%, 31%, and 31% for small, medium, large, and very large rooms, respectively.

IV. Conclusion

Sensors are prone to breaking, thus performance of a sensor-based application can be heavily impacted by missing sensors. We propose a new robustness-aware and probabilistic sensor placement method to maintain the coverage of a sensor field even with missing sensors. Our method increases the robustness of a sensor-based system by up to 50% compared to a coverage-only approach, with up to 201% higher probability of monitoring the entire area, even with broken sensors.

Acknowledgement

This work has been funded in part by NSF, with award numbers #1830331, #1911095, #1826967, #1730158, and #1527034. It was also partially supported by SRC task #2805.001.

REFERENCES

[1] A. T. Murray, K. Kim, J. W. Davis, R. Machiraju, and R. Parent, "Coverage optimization to support security monitoring," *Computers, Environment and Urban Systems*, vol. 31, no. 2, pp. 133–147, 2007.

[2] H. Xu, J. Zhu, and B. Wang, "On the deployment of a connected sensor network for confident information coverage," *Sensors*, vol. 15, no. 5, pp. 11277–11294, 2015.

[3] M. P. Fanti, M. Roccotelli, G. Faraut, and J.-J. Lesage, "Smart placement of motion sensors in a home environment," in *2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pp. 894–899, IEEE, 2017.

[4] O. Moh'd Alia and A. Al-Ajouri, "Maximizing wireless sensor network coverage with minimum cost using harmony search algorithm," *IEEE Sensors Journal*, vol. 17, no. 3, pp. 882–896, 2016.

[5] C. Yang and K.-W. Chin, "On nodes placement in energy harvesting wireless sensor networks for coverage and connectivity," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 1, pp. 27–36, 2016.

[6] Y. Pei and M. W. Mutka, "Joint bandwidth-aware relay placement and routing in heterogeneous wireless networks," in *2011 IEEE 17th International Conference on Parallel and Distributed Systems*, pp. 420–427, IEEE, 2011.

[7] A. Maheshwari and N. Chand, "A survey on wireless sensor networks coverage problems," in *Proceedings of 2nd International Conference on Communication, Computing and Networking*, pp. 153–164, Springer, 2019.

[8] B. Wang, "Coverage problems in sensor networks: A survey," *ACM Computing Surveys (CSUR)*, vol. 43, no. 4, pp. 1–53, 2011.

[9] F. Y. Lin and P.-L. Chiu, "A near-optimal sensor placement algorithm to achieve complete coverage-discrimination in sensor networks," *IEEE Communications Letters*, vol. 9, no. 1, pp. 43–45, 2005.

[10] I. Vlasenko, I. Nikolaidis, and E. Stroulia, "The smart-condo: Optimizing sensor placement for indoor localization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 3, pp. 436–453, 2014.

[11] M. P. Fanti, G. Faraut, J.-J. Lesage, and M. Roccotelli, "An integrated framework for binary sensor placement and inhabitants location tracking," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 1, pp. 154–160, 2016.

[12] L. Sela and S. Amin, "Robust sensor placement for pipeline monitoring: Mixed integer and greedy optimization," *Advanced Engineering Informatics*, vol. 36, pp. 55–63, 2018.

[13] A. N. Njoya, C. Thron, J. Barry, W. Abdou, E. Tonye, N. S. L. Konje, and A. Dipanda, "Efficient scalable sensor node placement algorithm for fixed target coverage applications of wireless sensor networks," *IET Wireless Sensor Systems*, vol. 7, no. 2, pp. 44–54, 2017.

[14] Y. Chae and D. N. Wilke, "Heuristic linear algebraic rank-variance formulation and solution approach for efficient sensor placement," *Engineering Structures*, vol. 153, pp. 717–731, 2017.

[15] J. A. Grant, A. Boukouvalas, R.-R. Griffiths, D. S. Leslie, S. Vakili, and E. M. De Cote, "Adaptive sensor placement for continuous spaces," *arXiv preprint arXiv:1905.06821*, 2019.

[16] R. Hou, Y. Xia, Q. Xia, and X. Zhou, "Genetic algorithm based optimal sensor placement for l1-regularized damage detection," *Structural Control and Health Monitoring*, vol. 26, no. 1, p. e2274, 2019.

[17] M. R. Senouci and A. Abdellaoui, "Efficient sensor placement heuristics," in *2017 IEEE International Conference on Communications (ICC)*, pp. 1–6, IEEE, 2017.

[18] C. Yang, K.-W. Chin, Y. Liu, J. Zhang, and T. He, "Robust targets coverage for energy harvesting wireless sensor networks," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 6, pp. 5884–5892, 2019.

[19] R. Mohan and B. de Jager, "Robust optimal sensor planning for occlusion handling in dynamic robotic environments," *IEEE Sensors Journal*, vol. 19, no. 11, pp. 4259–4270, 2019.

[20] M. Erdelj, N. Mitton, and T. Razafindralambo, "Robust wireless sensor network deployment," 2016.

[21] K. Vu and R. Zheng, "Robust coverage under uncertainty in wireless sensor networks," in *2011 Proceedings IEEE INFOCOM*, pp. 2015–2023, IEEE, 2011.

[22] K. Xu, G. Takahara, and H. Hassanein, "On the robustness of grid-based deployment in wireless sensor networks," in *Proceedings of the 2006 international conference on Wireless communications and mobile computing*, pp. 1183–1188, 2006.

[23] O. Karasakal and E. K. Karasakal, "A maximal covering location model in the presence of partial coverage," *Computers & Operations Research*, vol. 31, no. 9, pp. 1515–1526, 2004.

[24] R. R. Brooks and S. S. Iyengar, *Multi-sensor fusion: fundamentals and applications with software*. Prentice-Hall, Inc., 1998.

[25] S. S. Dhillon, K. Chakrabarty, and S. S. Iyengar, "Sensor placement for grid coverage under imprecise detections," in *Proceedings of the Fifth International Conference on Information Fusion. FUSION 2002.(IEEE Cat. No. 02EX5997)*, vol. 2, pp. 1581–1587, IEEE, 2002.

[26] R. T. Marler and J. S. Arora, "The weighted sum method for multi-objective optimization: new insights," *Structural and multidisciplinary optimization*, vol. 41, no. 6, pp. 853–862, 2010.

[27] C. U. Manual, "Ibm ilog cplex optimization studio," *Version*, vol. 12, pp. 1987–2018, 1987.

[28] S. Shelke and B. Aksanli, "Static and dynamic activity detection with ambient sensors in smart spaces," *Sensors*, vol. 19, no. 4, p. 804, 2019.

[29] M. I. Hamakareem, "Minimum height and size standards for rooms in buildings." URL: https://theconstructor.org/building/rooms-minimum-height-size-standards/5116/, Nov 2018.

[30] İ. K. Altınel, N. Aras, E. Güney, and C. Ersoy, "Binary integer programming formulation and heuristics for differentiated coverage in heterogeneous sensor networks," *Computer Networks*, vol. 52, no. 12, pp. 2419–2431, 2008.